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Analysis of building structures by replacement sandwich beams

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Abstract

In this paper replacement beams of building structures are developed, and the stiffnesses of the replacement beams are derived. The analysis is robust and can be used for slender and wide structures consisting of frames, trusses, shear walls, or coupled shear walls. The utility of the derived replacement beam is demonstrated through the examples of the in plane and flexural–torsional buckling and vibration analyses of high-rise buildings.

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1. Introduction

Analysis of high-rise building structures stiffened by shear walls, trusses, coupled shear walls, and frames requires time consuming numerical computations. The designer may be well served by approximate methods, which (i) can be used in the preliminary design when some of the structural dimensions are not yet known, (ii) can verify the results of the more advanced numerical calculation, and, last but not least (iii) can shed light on the behavior of the structure which may lead to a better design.

One of the most widely used approximate calculations is based on the "continuum method" (Zalka, 2000b), when the stiffened building structure is replaced by a (continuous) beam.

The simplest replacement beam is a *thin-walled beam*, characterized by the bending stiffnesses (D_{0yy}, D_{0zz}, D_{0yz}), the warping stiffness (D_ω) and the torsional stiffness (D_t). When only a plane problem is considered, torsion is excluded, and the only parameter that plays a role is the bending stiffness $D_0 = D_{0yy}$ in the x–z symmetry plane. This model is adequate only for solid and slender shear walls.

When a truss is loaded laterally, it may show, depending on the stiffnesses of the elements, bending (flexural) deformation, shear deformation, or the mixture of those (Fig. 1).

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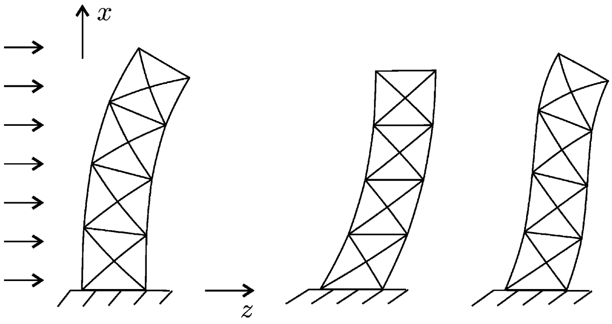


Fig. 1. Flexural deformation, shear deformation, and mixed deformation.

Hence the shear deformation must be included and the replacement continuum is a *Timoshenko-beam* (Timoshenko and Gere, 1961), characterized by the bending ($D_0 = D_{0yy}$) and the shear stiffnesses ($S = S_{zz}$) in the x–z plane. The bending and the shear stiffnesses of typical structures are given e.g. in Timoshenko and Gere (1961), Stafford Smith and Coull (1991) and Köpecsiri and Kollár (1999a,b) and are listed in Table 1.

Neither a thin-walled beam, nor a Timoshenko-beam is adequate to characterize a frame, or coupled shear walls. The replacement beam can be obtained by "smearing out" the beams of the frame along the height, and thus we arrive at the model shown in Fig. 2, which is a *sandwich beam* (Skattum, 1971).

The stiffnesses of the replacement sandwich beam are also included in Table 1 (Hegedűs and Kollár, 1999; Szerémi, 1978; Csonka, 1965; Beck, 1962), D_0 is the global bending stiffness, D_l is the local bending stiffness, and S is the shear stiffness).

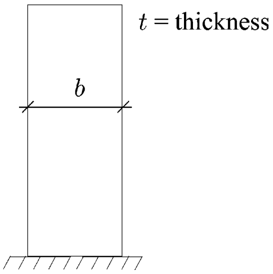
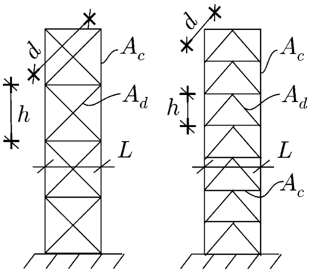
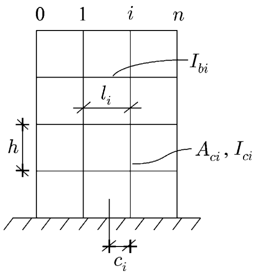
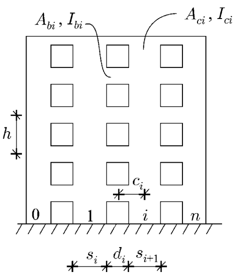
It is important to note that a sandwich beam with stiffnesses D_0, D_l, and S is equivalent to a Timoshenko-beam (with stiffnesses D_0 and S) which is supported laterally by a beam with bending stiffness D_l (Fig. 2). Hence, if we set the stiffness D_l of a sandwich beam equal to zero we obtain a Timoshenko-beam with stiffnesses D_0 and S.

Continuum models were developed by several authors and it was applied successfully for building structures subjected to wind loads (Csonka, 1965; Stafford Smith and Coull, 1991; Szerémi, 1978; Zalka, 2000a,b), earthquakes (Basu, 1983; Basu et al., 1984; Köpecsiri and Kollár, 1999a,b; Kollár, 1991; Stafford Smith and Coull, 1991), in the dynamic analysis (Ng and Kuang, 2000; Rosman, 1973, 1974; Skattum, 1971; Zalka, 1993, 1994, 2000a,b, 2001), and in the stability analysis (Hegedűs and Kollár, 1999; Li, 2000; Rosman, 1973, 1974; Stafford Smith and Coull, 1991; Zalka, 1998, 1999, 2000a,b; Zalka and Armer, 1992).

However, there are two important problems to be solved:

(i) As we stated before the replacement beam of a single lateral load-resisting subsystem (truss, frame, shear wall, etc.) is given in the literature (see Table 1). When there are several parallel lateral load-resisting subsystems which are connected horizontally along the height the question arises: how can they be replaced by only one replacement beam? We find answers only for the following special cases in the literature: (a) When each lateral load-resisting subsystem is a solid wall (their shear deformation is neglected) the replacement beam is a beam which undergoes bending deformation only, and its bending stiffness is the sum of the bending stiffnesses of the individual walls. (b) When there are frames which can be modeled as beams undergo shear deformation only and solid walls undergo bending deformation only, the replacement beam has two stiffnesses (with the sandwich notation D_l and S, while D_0 is infinite), the bending stiffness is the sum of the bending stiffnesses of the walls, while the shear stiffness is the sum of the shear stiffnesses of the frames. However, when any of the lateral load-resisting subsystem undergoes both bending and shear deformation (which is the case of trusses, coupled shear walls, tall frames, and for wide walls) it can be shown that simple summation ($S = \sum S_k$, $D_0 = \sum D_{0k}$, $D_l = \sum D_{lk}$) may result in a structure which is stiffer

Table 1
Replacement stiffnesses of different lateral load-resisting subsystems of high-rise buildings

Structure		Replacement continuum stiffnesses
Wall		Timoshenko-beam $D_0 = EI \qquad I = \frac{b^3 t}{12}$ $S = \frac{AG}{\rho} = \frac{AG}{1.2} \qquad A = bt$
Trusses		Timoshenko-beam $D_0 = \frac{1}{2} E A_c L^2 \qquad D_0 = \frac{1}{2} E A_c L^2$ $S = \frac{2EhL^2 A_d}{d^3} \qquad S = \frac{2Eh}{\frac{2d^3}{L^2 A_d} + \frac{L}{4A_b}}$ The shear stiffness of trusses with other type of bracing can be found in the literature.
Frame		Sandwich beam $D_1 = \sum_{i=0}^{n} EI_{ci}$ $D_0 = \sum_{i=0}^{n} EA_{ci} c_i^2$ $S = (S_b^{-1} + S_c^{-1})^{-1}$ $S_b = \sum_{i=1}^{n} \frac{12EI_{bi}}{l_i h}$, $S_c = \sum_{i=0}^{n} \frac{12EI_{ci}}{h^2}$
Coupled shear wall		Sandwich beam $D_1 = \sum_{i=0}^{n} EI_{ci}$ $D_0 = \sum_{i=0}^{n} EA_{ci} c_i^2$ $S = (S_b^{-1} + S_c^{-1})^{-1}$ $S_b = \sum_{i=1}^{n} \frac{6EI_{bi}[(d_i+s_i)^2 + (d_i+s_{i+1})^2]}{d_i^3 h \left(1 + \frac{12\rho EI_{bi}}{G d_i^2 A_{bi}}\right)}$ $S_c = \sum_{i=0}^{n} \frac{12EI_{ci}}{h^2}$

by orders of magnitudes than the “real” structure. We will show in Section 5 how the replacement stiffnesses of the building should be calculated.

(ii) As an example let us consider a structure the cross-section of which is shown in Fig. 4. When the structure is subjected to torsion, in the two parallel trusses both shear and bending deformations occur. The

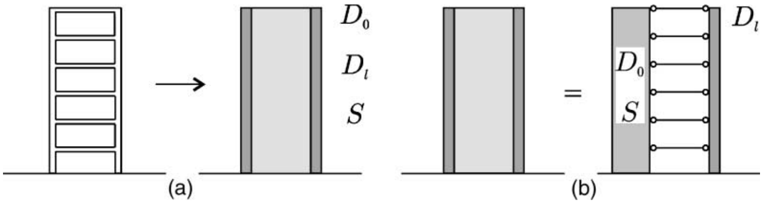


Fig. 2. Replacement beam of a frame (a), the sandwich beam is equivalent to a Timoshenko-beam supported by a beam with bending deformation only (b).

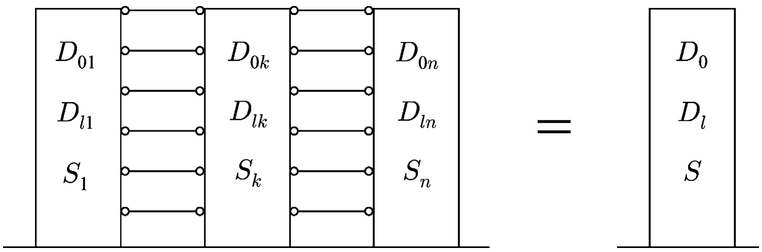


Fig. 3. Parallel lateral load-resisting subsystems and the replacement sandwich beam.

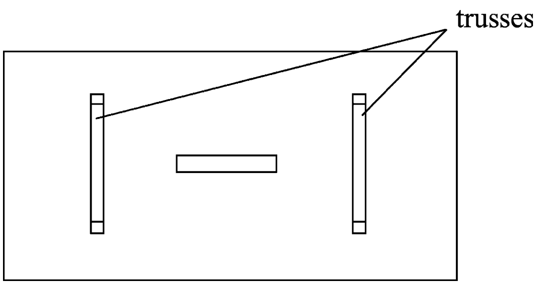


Fig. 4. Plan of a building stiffened by three lateral load-resisting subsystems.

classical (Vlasov) theory of beams does not include the shear deformation in torsion with warping and, hence, its application may significantly overestimate the torsional stiffness of the structure. This problem, for arbitrary arrangements of the walls, will be addressed in Section 6.

2. Problem statement

We consider a building structure, that consists of an arbitrary combination of lateral load-resisting subsystems, i.e. shear walls, coupled shear walls, frames, trusses and cores. The arrangement of the stiffening system is either symmetrical or arbitrary (Fig. 5).

Our aim is to obtain a replacement beam model and its replacement stiffnesses which can be used in the wind, earthquake, or stability analyses of the building structure.

The application of the continuum method for the earthquake analysis of building structures can be found in Potzta (2002). The method is also applicable for replacement beams with varying cross-section using a simple approximation given in Potzta and Kollár (1999).

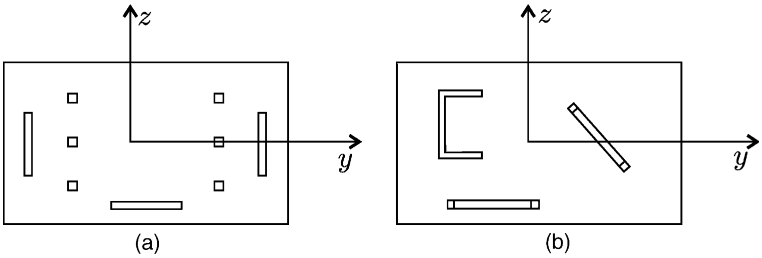


Fig. 5. Symmetrical (a) and unsymmetrical (b) arrangements of the lateral load-resisting subsystems.

3. Basic assumptions

We assume that the material behaves in a linearly elastic manner.

The floors are considered to be rigid in their plane and they transfer only horizontal forces but no bending or vertical forces to the lateral load-resisting subsystems. In addition, we assume that the floors connect the stiffening system "continuously", hence each cross-section of the building remains undeformed in the horizontal plane during loading.

4. Error in the analysis

The total error of the suggested method comes from two sources:

(i) Each lateral load-resisting subsystem is replaced by a continuous cantilever beam. The accuracy of this replacement can be found in the literature (Stafford Smith and Coull, 1991; Zalka and Armer, 1992; Zalka, 2000a,b). It is recommended that the building must have at least four stories (Zalka, 2000b).

(ii) These beams are then replaced by a single replacement sandwich beam. The error was investigated numerically in Potzta (2002) and it is illustrated in Section 8. It was found that this approximation results an error which is about 5%.

5. Plane problem

In this section we consider symmetrical stiffening systems which deform only in the plane of symmetry (x–z plane). The floors connect the lateral load-resisting subsystems continuously, hence the horizontal displacements of the lateral load-resisting subsystems are identical.

5.1. One lateral load-resisting subsystem

As it was stated in the Introduction, the replacement beam of a lateral load-resisting subsystem is a sandwich beam. The replacement stiffnesses are summarized in Table 1.

For latter use we define the strain energy of a sandwich beam (Allen, 1969):

$$U = U_{\mathrm{T}} + U_{\mathrm{l}}, \tag{1}$$

where

$$U_{\mathrm{T}} = \frac{1}{2}\int (S\gamma^2 + D_0(\chi')^2)\,\mathrm{d}x, \quad U_{\mathrm{l}} = \frac{1}{2}\int D_{\mathrm{l}}(w'')^2\,\mathrm{d}x. \tag{2}$$

Here U_T and U_l are the strain energies of a Timoshenko-beam (stiffnesses D_0 and S) and of a beam with bending deformation only (stiffness D_l), respectively. w is the displacement in the x–z plane, χ is the rotation of the cross-section in the x–z plane, and γ is the shear strain. Prime denotes derivative with respect to x. χ and γ are related to the displacement w by

$$w' = \chi + \gamma. \tag{3}$$

It can be shown (Hegedűs and Kollár, 1999) that in a sandwich beam:

$$\gamma = -\frac{D_0}{S}\chi''. \tag{4}$$

5.2. Several lateral load-resisting subsystems

In this section we consider n lateral load-resisting subsystems (Fig. 3). The kth element has the stiffnesses D_{0k}, D_{lk}, and S_k. The stiffnesses of the beam which replaces the n lateral load-resisting subsystems are denoted by D_0, D_l, and S.

We determine the replacement stiffnesses by applying a sinusoidal displacement on the stiffening system which is caused by a sinusoidal horizontal load (Fig. 6).

Then we equate the sum of the strain energies of the individual lateral load-resisting subsystems to the strain energy of the replacement wall. Hence (see Eq. (1)) we write

$$\frac{1}{2}\int \left(S\gamma^2 + D_0(\chi')^2 + D_l(w'')^2\right) dx = \frac{1}{2}\int \sum_{k=1}^{n}\left(S_k\gamma_k^2 + D_{0k}(\chi_k')^2 + D_{lk}(w_k'')^2\right) dx, \tag{5}$$

w_k, γ_k, and χ_k are the displacement, the shear strain, and the rotation of cross-section of the kth lateral load-resisting subsystem, and w, γ, and χ are the displacement, the shear strain, and the rotation of cross-section of the replacement beam, respectively. The horizontal displacements of the lateral load-resisting subsystems are identical, hence we can write

$$w_1 = w_2 = \cdots = w_n = w. \tag{6}$$

We apply the displacements

$$w = w_0 \sin\frac{\pi}{l}x, \quad \chi = \chi_0 \cos\frac{\pi}{l}x \tag{7}$$

on the beam. Eqs. (3), (4), and (7) yield

$$\chi = \frac{\frac{\pi}{l}}{1 + \frac{\pi^2}{l^2}\frac{D_0}{S}} w_0 \cos\frac{\pi}{l}x, \quad \gamma = \frac{\pi^2}{l^2}\frac{D_0}{S}\chi. \tag{8}$$

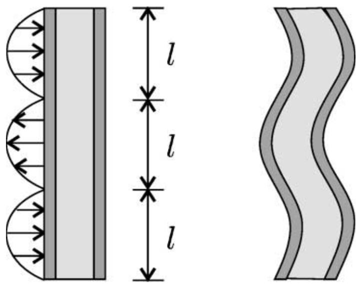


Fig. 6. The load and the deformations of a sandwich beam subjected to a sinusoidal load.

By introducing Eqs. (6)–(8) into Eq. (5) and performing the integration between 0 and l we obtain

$$D_1 + \frac{D_0}{1+\frac{\pi^2 D_0}{l^2 S}} = \sum_{k=1}^{n}\left(D_{1k} + \frac{D_{0k}}{1+\frac{\pi^2 D_{0k}}{l^2 S_k}}\right). \tag{9}$$

When l is large γ is small (see Eq. (8)) and Eq. (9) results in

$$D_1 + D_0 = \sum_{k=1}^{n}(D_{1k} + D_{0k}). \tag{10}$$

As a consequence we may state that for large l the replacement beam is a beam which undergoes bending deformation only.

The Taylor series expansion of the function $1/(1+\pi^2 D_0/(l^2 S))$ with respect to $1/l^2$ about $1/l_0^2$ is

$$\begin{aligned}\frac{1}{1+\frac{\pi^2 D_0}{l^2 S}} &= \frac{1}{1+\frac{\pi^2 D_0}{l_0^2 S}} - \frac{\frac{\pi^2 D_0}{S}}{\left(1+\frac{\pi^2 D_0}{l_0^2 S}\right)^2}\left(\frac{1}{l^2}-\frac{1}{l_0^2}\right) + \frac{\left(\frac{\pi^2 D_0}{S}\right)^2}{\left(1+\frac{\pi^2 D_0}{l_0^2 S}\right)^3}\left(\frac{1}{l^2}-\frac{1}{l_0^2}\right)^2 - \cdots \\ &= \sum_{i=0}^{\infty}\frac{\left(-\frac{\pi^2 D_0}{S}\right)^i}{\left(1+\frac{\pi^2 D_0}{l_0^2 S}\right)^{i+1}}\left(\frac{1}{l^2}-\frac{1}{l_0^2}\right)^i.\end{aligned} \tag{11}$$

Introducing Eq. (11) into Eq. (9) yields

$$D_1 + \sum_{i=0}^{\infty}\frac{D_0\left(-\frac{\pi^2 D_0}{S}\right)^i}{\left(1+\frac{\pi^2 D_0}{l_0^2 S}\right)^{i+1}}\left(\frac{1}{l^2}-\frac{1}{l_0^2}\right)^i = \sum_{k=1}^{n}\left(D_{1k} + \sum_{i=0}^{\infty}\frac{D_{0k}\left(-\frac{\pi^2 D_{0k}}{S_k}\right)^i}{\left(1+\frac{\pi^2 D_{0k}}{l_0^2 S_k}\right)^{i+1}}\left(\frac{1}{l^2}-\frac{1}{l_0^2}\right)^i\right).$$

When l is close to l_0 the terms multiplied by $(1/l^2 - 1/l_0^2)^i$ $(i = 1, 2, \ldots)$ vanish and we have

$$D_1 + \frac{D_0}{1+\frac{\pi^2 D_0}{l_0^2 S}} = \sum_{k=1}^{n}\left(D_{1k} + \frac{D_{0k}}{1+\frac{\pi^2 D_{0k}}{l_0^2 S_k}}\right). \tag{12}$$

To obtain a good agreement between the replacement beam and the structure the first three terms in the series are considered. By equating the first term in the series we obtain Eq. (12), while from the second and third terms we have

$$\frac{D_0}{\left(1+\frac{\pi^2 D_0}{l_0^2 S}\right)^2}\frac{\pi^2 D_0}{S} = \sum_{k=1}^{n}\frac{D_{0k}}{\left(1+\frac{\pi^2 D_{0k}}{l_0^2 S_k}\right)^2}\frac{\pi^2 D_{0k}}{S_k}, \tag{13}$$

$$\frac{D_0}{\left(1+\frac{\pi^2 D_0}{l_0^2 S}\right)^3}\left(\frac{\pi^2 D_0}{S}\right)^2 = \sum_{k=1}^{n}\frac{D_{0k}}{\left(1+\frac{\pi^2 D_{0k}}{l_0^2 S_k}\right)^3}\left(\frac{\pi^2 D_{0k}}{S_k}\right)^2. \tag{14}$$

Eqs. (12)–(14) can be rearranged to yield the replacement stiffnesses of the beam:

$$S = \pi^2\frac{B^3}{C^2}, \quad D_0 = \frac{1}{\frac{C}{B^2} - \frac{1}{l_0^2}\frac{C^2}{B^3}}, \quad D_1 = A - \frac{B^2}{C}, \tag{15}$$

where

$$A=\sum_{k=1}^{n}\left(\frac{D_{0k}}{1+\frac{\pi^2 D_{0k}}{l_0^2 S_k}}+D_{1k}\right),$$
$$B=\sum_{k=1}^{n}\frac{D_{0k}}{\left(1+\frac{\pi^2 D_{0k}}{l_0^2 S_k}\right)^2}\frac{\pi^2 D_{0k}}{S_k}, \tag{16}$$
$$C=\sum_{k=1}^{n}\frac{D_{0k}}{\left(1+\frac{\pi^2 D_{0k}}{l_0^2 S_k}\right)^3}\left(\frac{\pi^2 D_{0k}}{S_k}\right)^2.$$

The choice of l_0 to obtain the "best" replacement stiffnesses will be discussed in Section 7.

6. Spatial problem

As it was stated in the Introduction, the beam undergoes both shear and bending (flexural) deformations in torsion. We adopt here the beam theory given in Kollár (2001a,b) (which was developed for composite beams and which is the simplification of Sun and Wu's theory (Wu and Sun, 1992)). Accordingly, the displacements of the beam are described by the vectors

$$\{u\}=\left\{\begin{matrix} v \\ w \\ \psi \end{matrix}\right\},\quad \{\chi\}=\left\{\begin{matrix} \chi_y \\ \chi_z \\ \vartheta_B \end{matrix}\right\}, \tag{17}$$

while the shear deformations are

$$\{\gamma\}=\left\{\begin{matrix} \gamma_y \\ \gamma_z \\ \vartheta_S \end{matrix}\right\}. \tag{18}$$

v and w are the displacements in the y and z directions, ψ is the rotation of the cross-section about the x axis, χ_y and χ_z are the rotations of the cross-section about the z and y axes, respectively. The total twist per unit length (ϑ) contains a shear type (ϑ_S) and a bending type (ϑ_B) deformation (Fig. 7):

$$\vartheta=\psi'=\vartheta_B+\vartheta_S.$$

(We note that in Vlasov's theory $\gamma_y=\gamma_z=\vartheta_S=0$ and $\{u\}'=\{\chi\}$.) The shear deformations are related to the displacements by (see Kollár, 2001a,b)

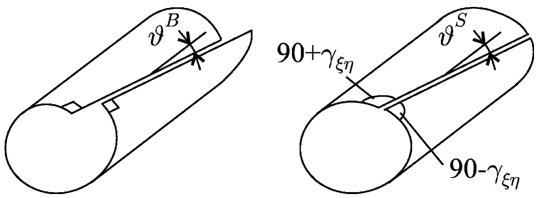


Fig. 7. Bending (ϑ_B) and shear (ϑ_S) deformation in torsion.

$$\{u\}' = \begin{Bmatrix} v' \\ w' \\ \psi' \end{Bmatrix} = \{\chi\} + \{\gamma\}. \tag{19}$$

The strain energy of this beam is

$$U_{\rm T} = \frac{1}{2}\int\left(\{\gamma\}^{\rm T}\begin{bmatrix} S_{yy} & S_{yz} & S_{y\omega} \\ S_{zy} & S_{zz} & S_{z\omega} \\ S_{\omega z} & S_{\omega y} & S_{\omega\omega}\end{bmatrix}\{\gamma\} + \{\chi\}'^{\rm T}\begin{bmatrix} D_{0zz} & D_{0zy} & D_{0z\omega} \\ D_{0yz} & D_{0yy} & D_{0y\omega} \\ D_{0\omega z} & D_{0\omega y} & D_{0\omega\omega}\end{bmatrix}\{\chi\}' + D_{\rm t}\vartheta^2\right){\rm d}x. \tag{20}$$

In this equation D_{0yy} $(= EI_{yy})$, D_{0yz} $(= EI_{yz})$, $D_{0zz}(= EI_{zz})$ are the bending stiffnesses, $D_{0\omega\omega}(= EI_{\omega})$ is the warping stiffness. ($D_{0\omega z}$ and $D_{0\omega y}$ are zero if the coordinate system is attached to the shear center see Section 6.3.) $[S]$ is the shear stiffness matrix and $D_{\rm t}$ $(= GI_{\rm t})$ is the torsional stiffness. $\{\ \}^{\rm T}$ denotes the transpose of vector $\{\ \}$.

The above beam theory is the generalization of the "Timoshenko-beam" theory for spatial problems. For our case, as in the plane problem, the local stiffnesses must also be included, and the strain energy becomes

$$U = U_{\rm T} + U_{\rm l}, \tag{21}$$

where $U_{\rm l}$ is the strain energy of a beam which undergoes bending deformation only:

$$U_{\rm l} = \frac{1}{2}\int\{u\}''^{\rm T}\begin{bmatrix} D_{{\rm l}zz} & D_{{\rm l}zy} & D_{{\rm l}z\omega} \\ D_{{\rm l}yz} & D_{{\rm l}yy} & D_{{\rm l}y\omega} \\ D_{{\rm l}\omega z} & D_{{\rm l}\omega y} & D_{{\rm l}\omega\omega}\end{bmatrix}\{u\}''\,{\rm d}x. \tag{22}$$

Here $\{u\}$ is the displacement vector (Eq. (17)), and $D_{{\rm l}ij}$ are the (local) bending stiffnesses.

6.1. One lateral load-resisting subsystem

First we consider a single lateral load-resisting subsystem (Fig. 8).

The stiffness matrices of this bracing element in the η–ζ coordinate system are $[D_0^{\eta-\zeta}]_k$, $[D_1^{\eta-\zeta}]_k$ and $[S^{\eta-\zeta}]_k$, and the torsional stiffness is $D_{{\rm t}k}^{\eta-\zeta}$. The transformation of the stiffnesses into the y–z coordinate system gives

$$\begin{aligned} [D_{\rm l}]_k &= [T]_k^{\rm T}[D_{\rm l}^{\eta-\zeta}]_k[T]_k, \\ [D_0]_k &= [T]_k^{\rm T}[D_0^{\eta-\zeta}]_k[T]_k, \\ [S]_k &= [T]_k^{\rm T}[S^{\eta-\zeta}]_k[T]_k, \end{aligned} \tag{23}$$

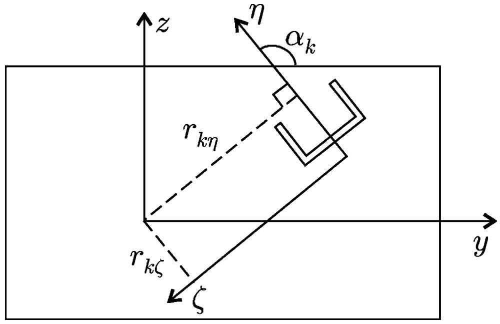


Fig. 8. Global coordinates (y, z) and the local coordinates (η, ζ) attached to the kth lateral load-resisting subsystem.

where

$$[T]_k = \begin{bmatrix} \cos\alpha_k & -\sin\alpha_k & r_{k\eta} \\ \sin\alpha_k & \cos\alpha_k & r_{k\zeta} \\ 0 & 0 & 1 \end{bmatrix}, \tag{24}$$

α_k is the angle between the axes η and y; $r_{k\eta}$ and $r_{k\varsigma}$ are the distances of the η and ζ axes from the origin (Fig. 8).

6.2. Several lateral load-resisting subsystems

To obtain the replacement stiffnesses of a stiffening system containing several lateral load-resisting subsystems we assume the displacements in the form of

$$\{u\} = \begin{Bmatrix} v_0 \\ w_0 \\ \psi_0 \end{Bmatrix} \sin\frac{\pi}{l}x, \quad \{\chi\} = \begin{bmatrix} \chi_{y0} \\ \chi_{z0} \\ \vartheta_{B0} \end{bmatrix} \cos\frac{\pi}{l}x, \tag{25}$$

and introduce them into the expression of the strain energy of the replacement beam (Eq. (21)) and into the sum of the strain energies of the individual elements. By equating them we obtain

$$\begin{aligned} &\frac{1}{2}\int \left(\{\gamma\}^{\mathrm{T}}[S]\{\gamma\} + \{\chi\}'^{\mathrm{T}}[D_0]\{\chi\}' + \{u\}''^{\mathrm{T}}[D_1]\{u\}'' + D_t\vartheta^2\right)\mathrm{d}x \\ &\quad = \frac{1}{2}\int \sum_{k=1}^{n}\left(\{\gamma_k\}^{\mathrm{T}}[S_k]\{\gamma_k\} + \{\chi_k\}'^{\mathrm{T}}[D_{0k}]\{\chi_k\}' + \{u_k\}''^{\mathrm{T}}[D_{1k}]\{u_k\}'' + D_{tk}\vartheta^2\right)\mathrm{d}x. \end{aligned} \tag{26}$$

Then we follow the same steps as in Section 5.2. The algebra is involved but straightforward and is not presented here. The results are

$$\begin{aligned} &[S] = \pi^2[B][C]^{-1}[B][C]^{-1}[B], \\ &[D_0] = [B][C]^{-1}[B]\left[[E] - \frac{1}{l_0^2}[B]^{-1}[C]\right]^{-1}, \\ &[D_1] = [A] - [B][C]^{-1}[B], \\ &D_t = \sum_{k=1}^{n} D_{tk}, \end{aligned} \tag{27}$$

where

$$\begin{aligned} [A] &= \sum_{k=1}^{n}\left(\left([E] + \frac{\pi^2}{l_0^2}[D_0]_k[S]_k^{-1}\right)^{-1}[D_0]_k + [D_1]_k\right), \\ [B] &= \sum_{k=1}^{n}\pi^2\left([E] + \frac{\pi^2}{l_0^2}[D_0]_k[S]_k^{-1}\right)^{-1}[D_0]_k[S]_k^{-1}\left([E] + \frac{\pi^2}{l_0^2}[D_0]_k[S]_k^{-1}\right)^{-1}[D_0]_k, \\ [C] &= \sum_{k=1}^{n}\pi^4\left([E] + \frac{\pi^2}{l_0^2}[D_0]_k[S]_k^{-1}\right)^{-1}[D_0]_k[S]_k^{-1} \\ &\quad \times\left([E] + \frac{\pi^2}{l_0^2}[D_0]_k[S]_k^{-1}\right)^{-1}[D_0]_k[S]_k^{-1}\left([E] + \frac{\pi^2}{l_0^2}[D_0]_k[S]_k^{-1}\right)^{-1}[D_0]_k. \end{aligned} \tag{28}$$

We observe that Eq. (27) reduces to Eq. (15) when $[D_0]$, $[D_1]$, and $[S]$ are replaced by D_0, D_1 and S.

6.3. Location of the replacement beam

The elements of the stiffness matrices $[D_0]$, $[D_l]$ and $[S]$ depends on the choice of the location of the axis of the replacement beam which passes through the origin of the coordinate system.

In the analysis the location of the axis can be chosen arbitrarily. (The stiffnesses, loads, coordinates of the mass center are influenced by the location, however the eigen frequency, buckling loads, internal forces are not.)

There is a special location of the origin called shear center (or more precisely the "bending deformation shear center" (Kollár, 2001a,b)). When the origin of the coordinate system is attached to the bending deformation shear center the stiffness matrix $[D_0]$ simplifies and $D_{0z\omega}$ and $D_{0y\omega}$ are zero. However, this choice of the origin simplifies the analysis only when the shear deformation is neglected (shear stiffnesses are infinite), because in this case the load applied at the shear center does not cause the twist of the building.

As a rule, when the beam undergoes both bending and shear deformations there is no such location and the beam may twist even if the load is applied at the bending deformation shear center.

However for a given load and boundary conditions we may define a location (which varies with the height) such a way that the load acting at this location do not cause the twist of the building.

When the building is symmetrical, the load which is applied in the symmetry plane does not cause twist, and hence it is practical (however not necessary) to place the axis of the beam at the symmetry plane.

7. Practical considerations

In Sections 5.2 and 6.2 we obtained a replacement beam whose stiffnesses depend on the choice of l_0. When l_0 is approximately equal to the variation of the load (see Fig. 6) the behavior of the replacement beam will be very close to the behavior of the structure. Consequently different replacement beams should be applied for different loading conditions.

The question arises, how should we choose l_0 to obtain the "best" replacement beam. We suggest for a few cases the values l_0 which are given in Fig. 9.

We note that there are practical cases when the stiffnesses of the replacement beam are not sensitive to the choice of l_0 and hence the same replacement beam can be used for different loading conditions.

When $D_{0i}/S_i \approx D_{0j}/S_j$ or $S_i l_0^2 \gg D_{0i}$ Eqs. (15) and (16) become

$$S = \frac{B^3}{C^2}, \tag{29}$$

$$D_0 = \frac{B^2}{C}, \tag{30}$$

$$D_l = \sum_{k=1}^{n} (D_{0k} + D_{lk}) - D_0, \tag{31}$$

where

$$B = \sum_{k=1}^{n} \frac{D_{0k}^2}{S_k}, \quad C = \sum_{k=1}^{n} \frac{D_{0k}^3}{S_k^2}. \tag{32}$$

Note that these equations are identical to Eqs. (15) and (16) when $l_0 \to \infty$.

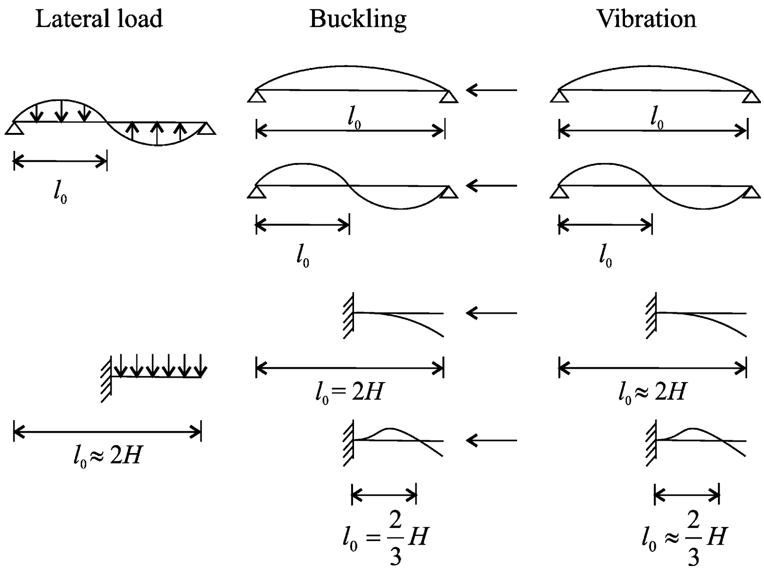


Fig. 9. The values of l_0 for lateral load, buckling and vibration.

When $S_i l_0^2 \ll D_{0i}$ Eqs. (15) and (16) become

$$S = \sum_{k=1}^{n} S_k, \quad D_0 = \infty, \quad D_1 = \sum_{k=1}^{n} D_{1k}. \tag{33}$$

These simplifications can be carried out also in case of spatial problems: When $[D_0]_i[S]_i^{-1} \approx [D_0]_j[S]_j^{-1}$or $[S]_i l_0^2 \gg [D_0]_i$ Eqs. (27) and (28) become

$$[S] = [B][C]^{-1}[B][C]^{-1}[B], \tag{34}$$

$$[D_0] = [B][C]^{-1}[B], \tag{35}$$

$$[D_1] = \sum_{k=1}^{n}([D_0]_k + [D_1]_k) - [D_0], \tag{36}$$

where

$$[B] = \sum_{k=1}^{n}[D_0]_k[S]_k^{-1}[D_0]_k, \quad [C] = \sum_{k=1}^{n}[D_0]_k[S]_k^{-1}[D_0]_k[S]_k^{-1}[D_0]_k. \tag{37}$$

For $[S]_i l_0^2 \ll [D_0]$ Eqs. (27) and (28) become

$$[S] = \sum_{k=1}^{n}[S]_k, \quad [D_0] = \infty, \quad [D_1] = \sum_{k=1}^{n}[D_1]_k. \tag{38}$$

$$H = 30 \times 3.05 = 91.5\,\text{m} \qquad M = 3.058 \times 10^5\,\text{kg/m}$$

$$E = 1.95 \times 10^{10}\,\text{N/m}^2 \qquad \Theta = m\frac{36^2 + 18^2}{12} = 4.128 \times 10^7\,\text{kgm}^2\text{/m}$$

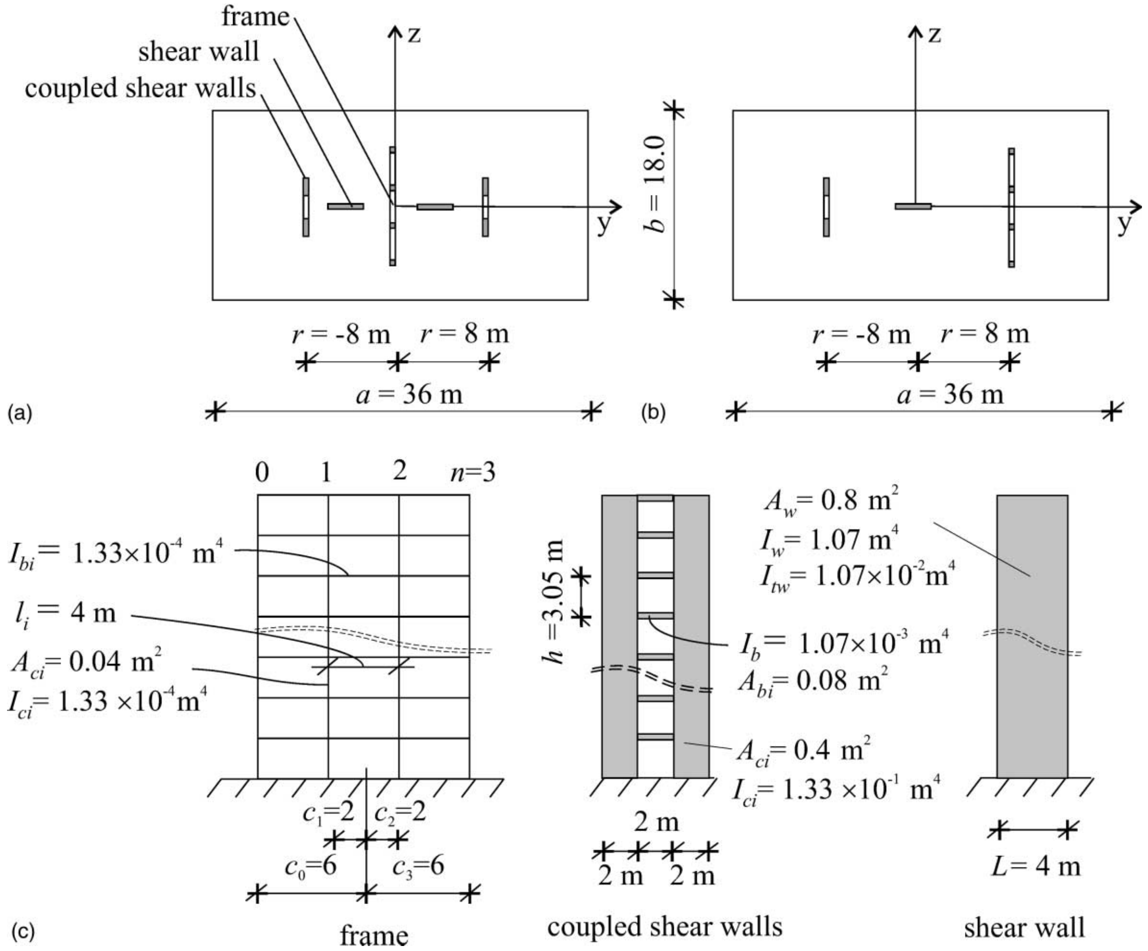


Fig. 10. Numerical example: (a) symmetrical structure, (b) unsymmetrical structure, (c) lateral load-resisting subsystems.

8. Numerical examples

To demonstrate the utility of the replacement beam we consider two simple examples shown in Fig. 10.

8.1. Doubly symmetrical structure

The structure is stiffened by two coupled shear walls, a frame, and two shear walls (Fig. 10a). We wish to determine in the x–z symmetry plane: (i) the circular frequencies of the structure when the mass is distributed uniformly along the floors, and (ii) the buckling loads when the load is applied at the top.

The replacement stiffnesses of the lateral load-resisting subsystems are (see Table 1):

$$
\begin{aligned}
D_1^{\mathrm{f}} &= \sum_{i=0}^{3} EI_{ci} = 4 \times 1.95 \times 10^{10} \times 1.33 \times 10^{-4} = 1.040 \times 10^{7}\ \mathrm{Nm}^2,\\
D_0^{\mathrm{f}} &= \sum_{i=0}^{3} EA_{ci}c_i^2 = 2 \times 1.95 \times 10^{10} \times 0.04 \times (2^2 + 6^2) = 6.240 \times 10^{10}\ \mathrm{Nm}^2,\\
S_b &= \sum_{i=1}^{3} \frac{12EI_{bi}}{l_i h} = 3 \times \frac{12 \times 1.95 \times 10^{10} \times 1.33 \times 10^{-4}}{4 \times 3.05} = 7.672 \times 10^{6}\ \mathrm{N},\\
S_c &= \sum_{i=0}^{3} \frac{12EI_{ci}}{h^2} = 4 \times \frac{12 \times 1.95 \times 10^{10} \times 1.33 \times 10^{-4}}{3.05^2} = 1.342 \times 10^{6}\ \mathrm{N},\\
S^{\mathrm{f}} &= (S_b^{-1} + S_c^{-1})^{-1} = 4.881 \times 10^{6}\ \mathrm{N},\\
D_1^{\mathrm{w}} &= 0,\\
D_0^{\mathrm{w}} &= EI_w = 1.95 \times 10^{10} \times 1.07 = 2.087 \times 10^{10}\ \mathrm{Nm}^2,\\
S^{\mathrm{w}} &= \frac{GA_w}{\rho} = \frac{\frac{1.95\times10^{10}}{2\times1.2} \times 0.8}{1.2} = 5.42 \times 10^{9}\ \mathrm{N},\\
D_t^{\mathrm{w}} &= GI_{tw} = \frac{1.95 \times 10^{10}}{2 \times 1.2} \times 1.07 \times 10^{-2} = 1.733 \times 10^{8}\ \mathrm{Nm}^2,\\
D_1^{\mathrm{c}} &= \sum_{i=0}^{1} EI_{ci} = 2 \times 1.95 \times 10^{10} \times 1.33 \times 10^{-1} = 5.200 \times 10^{9}\ \mathrm{Nm}^2,\\
D_0^{\mathrm{c}} &= \sum_{i=0}^{1} EA_{ci}c_i^2 = 2 \times 1.95 \times 10^{10} \times 0.4 \times 2^2 = 6.240 \times 10^{10}\ \mathrm{Nm}^2,\\
S_b &= \frac{6EI_b[(d+s_1)^2 + (d+s_2)^2]}{d^3h\left(1 + \frac{12\rho EI_b}{Gd^2A_b}\right)} = \frac{6 \times 1.95 \times 10^{10} \times 1.07 \times 10^{-3} \times 2 \times (2+2)^2}{2^3 \times 3.05\left(1 + \frac{12\times1.2\times1.95\times10^{10}\times1.07\times10^{-3}}{1.95\times10^{10}/2.4\times2^2\times0.08}\right)}\\
&= 1.468 \times 10^{8}\ \mathrm{N},\\
S_c &= \sum_{i=0}^{1} \frac{12EI_{ci}}{h^2} = 2 \times \frac{12 \times 1.95 \times 10^{10} \times 1.33 \times 10^{-1}}{3.05^2} = 6.708 \times 10^{9}\ \mathrm{N},\\
S^{\mathrm{c}} &= (S_b^{-1} + S_c^{-1})^{-1} = 1.436 \times 10^{8}\ \mathrm{N},
\end{aligned}
\tag{39}
$$

where superscript f, w, and c refers to the frame, to the walls, to the coupled shear walls, respectively.

The structure is symmetrical, thus the replacement stiffnesses of the structure can be calculated from Eqs. (15) and (16). We choose $l_{01} = 2H$, $l_{02} = \frac{2}{3}H$, and $l_{03} = \frac{2}{5}H$ in the analysis of the first, second and third mode of vibration, respectively (see Fig. 9). The results are given in Table 2.

Using the replacement stiffnesses we approximate the circular frequencies, ω_i (in the x–z symmetry plane) as follows (Potzta, 2002):

$$
\omega_i^2 = \left(\frac{1}{(\omega_i^{B_0})^2} + \frac{1}{(\omega_i^{S})^2}\right)^{-1} + (\omega_i^{B_1})^2 = \left(\frac{1}{\mu_{Bi}^2 \frac{D_{0i}}{mH^4}} + \frac{1}{\mu_{Si}^2 \frac{S_i}{mH^2}}\right)^{-1} + \mu_{Bi}^2 \frac{D_{1i}}{mH^4},
\tag{40}
$$

where μ_{Bi} and μ_{Si} for the first three modes are given in Table 3, the mass, M, and the total height of the building, H are given in Fig. 10. The approximate value of the circular frequencies (Eq. (40)) and the results

Table 2
Replacement stiffnesses of the symmetrical structure given in Fig. 10

Mode	$i = 1$	$i = 2$	$i = 3$
l_{0i}	$2H$	$\frac{2}{3}H$	$\frac{2}{5}H$
A ($\times 10^{11}$ kN m^2)	1.341	0.7018	0.4077
B ($\times 10^{14}$ kN m^4)	7.669	1.220	0.3119
C ($\times 10^{17}$ kN m^6)	107.6	2.535	0.3120
D_{0i} ($\times 10^{10}$ kN m^2)	9.406	13.29	13.08
D_{li} ($\times 10^{10}$ kN m^2)	7.950	1.149	1.047
S_i ($\times 10^7$ kN)	3.841	27.87	29.06

Table 3
The values of the multiplier, μ_i for the calculation of the circular frequencies

Deformation	Mode		
	1	2	3
Bending (ν_{B1})	3.52	22.03	61.7
Shear (ν_{S1})	0.5π	1.5π	2.5π

Table 4
Circular frequencies (1/s) of the symmetrical structure (Fig. 10) using a finite element program (ETABS), and the results of four different approximations

Mode	ETABS	Sandwich beam	Summation of stiffnesses	Thin-walled beam	Timoshenko-beam
1	0.2649	0.2607	0.290	0.249	0.536
2	1.206	1.265	1.349	1.168	1.663
3	2.708	2.690	2.767	2.32	2.980

Table 5
Errors in the circular frequencies of the four approximations

Mode	Sandwich beam	Summation of stiffnesses	Thin-walled beam	Timoshenko-beam
1	−1.59%	9.48%	−6.00%	102.34%
2	4.89%	11.86%	−3.15%	37.89%
3	−0.66%	2.18%	−14.33%	10.04%

of a finite element calculation (using the ETABS program) are summarized in the second and third columns of Table 4, respectively. The maximum error is less then 5% (Table 5).

For comparison we calculated the circular frequencies also with the aid of other replacement beam models. The fourth column of Table 4 shows the results in case of simple summation of the stiffnesses of lateral load-resisting subsystems (each replaced by a sandwich beam). The last two columns of Table 4 present the results when the lateral load-resisting subsystems are replaced by thin-walled beams and by Timoshenko-beams, respectively (the replacement stiffnesses are calculated by Eqs. (15) and (16)). The errors are given in Table 5.

The buckling loads of the structure (when the load is applied at the top) also can be approximated using the replacement stiffnesses (Kollár, 2001a,b). Buckling loads in the x–z symmetry plane are

$$N_{\mathrm{cr}}^1 = \left(\left(\frac{\pi^2 D_{01}}{(2H)^2} \right)^{-1} + \frac{1}{S_1} \right)^{-1} + \frac{\pi^2 D_{11}}{(2H)^2} = 3.953 \times 10^4 \text{ kN},$$

$$N_{\mathrm{cr}}^2 = \left(\left(\frac{\pi^2 D_{02}}{\left(\frac{2}{3}H\right)^2} \right)^{-1} + \frac{1}{S_2} \right)^{-1} + \frac{\pi^2 D_{12}}{\left(\frac{2}{3}H\right)^2} = 1.861 \times 10^5 \text{ kN}, \quad (41)$$

$$N_{\mathrm{cr}}^3 = \left(\left(\frac{\pi^2 D_{03}}{\left(\frac{2}{5}H\right)^2} \right)^{-1} + \frac{1}{S_3} \right)^{-1} + \frac{\pi^2 D_{13}}{\left(\frac{2}{5}H\right)^2} = 3.004 \times 10^5 \text{ kN}.$$

These results are identical to the theoretical values calculated by the equations of Hegedűs and Kollár (1999).

The buckling loads were also calculated with the aid of other replacement beam models. The fourth column of Table 6 shows the results in case of simple summation of the stiffnesses of lateral load-resisting subsystems (each replaced by a sandwich beam). The last two columns of Table 6 present the results when the lateral load-resisting subsystems are replaced by thin-walled beams and by Timoshenko-beams, respectively (the replacement stiffnesses are calculated by Eqs. (15) and (16)). The errors given in Table 7 shows that—as a rule—only the accuracy of the replacement sandwich beam is adequate for practical purposes.

8.2. Unsymmetrical structure

The geometrical and material properties of the structure are given in Fig. 10b. The building is stiffened by a shear wall, coupled shear walls, and a frame as shown in Fig. 10b. We wish to determine the circular frequencies of the first three modes of vibration. The lateral load-resisting subsystems of the unsymmetrical structure are identical to those of the symmetrical structure (Section 8.1), the replacement stiffnesses are given by Eq. (39). The structure has one plane of symmetry (x–y). The structure vibrates either in the plane of symmetry (Section 5) or spatial, lateral–torsional vibration occurs (Section 6).

In the symmetry plane the circular frequencies can be calculated from Eq. (40) independently of the spatial vibration modes. The results for the first three modes are given in Table 8.

Table 6
Theoretical values of buckling loads (kN) of the symmetrical structure (Fig. 10), and the results of four different approximations

Mode	Exact	Sandwich beam	Summation of stiffnesses	Thin-walled beam	Timoshenko-beam
1	3.953×10^4	3.953×10^4	4.947×10^4	3.646×10^4	2.952×10^4
2	1.861×10^4	1.861×10^4	2.115×10^4	1.585×10^4	3.197×10^4
3	3.004×10^4	3.004×10^4	3.178×10^4	2.237×10^4	3.688×10^4

Table 7
Errors in the buckling loads of the four approximations

Mode	Sandwich beam	Summation of stiffnesses	Thin-walled beam	Timoshenko-beam
1	0%	25.15%	−7.77%	−25.32%
2	0%	13.65%	−14.83%	71.79%
3	0%	5.79%	−25.53%	22.77%

Table 8
Comparison of the numerical and approximate results for the circular frequencies of the unsymmetrical structure given in Fig. 10

Mode	Direction	Approximation	ETABS	Error (%)
1	Spatial vibration	0.07834	0.07272	7.73
1	x–y plane	0.1095	0.1095	0.00
1	Spatial vibration	0.2187	0.2263	−3.36
2	Spatial vibration	0.2459	0.2202	11.67
2	x–y plane	0.6828	0.6813	0.22
2	Spatial vibration	1.077	1.028	4.77
3	Spatial vibration	0.4154	0.3919	6.00
3	x–y plane	1.895	1.887	0.24
3	Spatial vibration	2.2974	2.314	−0.72

To calculate the circular frequencies of the coupled vibration modes first we determine the replacement stiffness matrices (Section 6). The stiffness matrices of the individual lateral load-resisting subsystems are

$$[D_0^{\eta-\zeta}]_1 = \begin{bmatrix} D_0^c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad [D_1^{\eta-\zeta}]_1 = \begin{bmatrix} D_1^c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad [S^{\eta-\zeta}]_1 = \begin{bmatrix} S^c & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix},$$
$$[D_0^{\eta-\zeta}]_2 = \begin{bmatrix} D_0^w & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad [D_1^{\eta-\zeta}]_2 = \begin{bmatrix} D_1^w & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad [S^{\eta-\zeta}]_2 = \begin{bmatrix} S^w & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}, \tag{42}$$
$$[D_0^{\eta-\zeta}]_3 = \begin{bmatrix} D_0^f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad [D_1^{\eta-\zeta}]_3 = \begin{bmatrix} D_1^f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad [S^{\eta-\zeta}]_3 = \begin{bmatrix} S^f & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix},$$
$$D_t = D_t^w.$$

(Because of numerical considerations we replaced the zero elements in the main diagonal of the shear stiffness matrices by a small element ε ($\varepsilon > 0$).)

The matrices of the transformation from the local (η–ζ) coordinate systems into the y–z coordinate system are

$$[T]_1 = [T]^c = \begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad [T]_2 = [T]^w = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad [T]_3 = [T]^f = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{43}$$

For $l_0 = 2H$ Eq. (28) yields

$$[A] = 10^{10} \times \begin{bmatrix} 7.362 & 0 & -37.93 \\ 0 & 2.0776 & 0 \\ -37.93 & 0 & 471.1 \end{bmatrix},$$
$$[B] = 10^{14} \times \begin{bmatrix} 5.556 & 0 & 10.89 \\ 0 & 7.865 \times 10^{-3} & 0 \\ 10.89 & 0 & 356.2 \end{bmatrix}, \tag{44}$$
$$[C] = 10^{18} \times \begin{bmatrix} 9.966 & 0 & 66.94 \\ 0 & 2.978 \times 10^{-5} & 0 \\ 66.94 & 0 & 637.8 \end{bmatrix}.$$

The stiffness matrices of the replacement beam are (Eq. (27))

$$D_0 = 10^{11} \times \begin{bmatrix} 1.248 & 0 & 0 \\ 0 & 0.2087 & 0 \\ 0 & 0 & 79.87 \end{bmatrix} \mathrm{kN\,m^2},$$

$$D_l = 10^{9} \times \begin{bmatrix} 5.210 & 0 & -41.52 \\ 0 & 0 & 0 \\ -41.52 & 0 & 333.5 \end{bmatrix} \mathrm{kN\,m^2}, \qquad (45)$$

$$S = 10^{8} \times \begin{bmatrix} 1.485 & 0 & -10.99 \\ 0 & 54.17 & 0 \\ -10.99 & 0 & 95.04 \end{bmatrix} \mathrm{kN},$$

$$D_t = 1.733 \times 10^8\ \mathrm{Nm^2}.$$

The circular frequencies of the lateral–torsional vibration modes can be determined as the eigenvalues of the following equation (Potzta, 2002):

$$\left[\left(\frac{H^4}{\mu_{Bi}^2}[D_0]^{-1} + \frac{H^2}{\mu_{Si}^2}[S]^{-1}\right)^{-1} + \frac{\mu_{Bi}^2}{H^4}[D_l] + \frac{\mu_{Si}^2}{H^2}[G] - \omega_{mi}^2 m[M]\right]\begin{Bmatrix} v_{0m} \\ w_{0m} \\ \psi_{0m} \end{Bmatrix} = 0, \qquad (46)$$

μ_{Bi} and μ_{Si} for the first three modes are given in Table 3, H is the total height of the building ($H = 91.5$ m), matrices $[M]$ and $[G]$ are

$$[M] = \begin{bmatrix} 1 & 0 & y_m \\ 0 & 1 & z_m \\ y_m & z_m & \frac{\Theta}{m} + y_m^2 + z_m^2 \end{bmatrix}, \quad [G] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & D_t \end{bmatrix}, \qquad (47)$$

m is the mass per unit height ($m = 3.058 \times 10^5$ kg/m), and Θ is the polar moment of mass (per unit height) about the mass center (Fig. 10), y_m, z_m are the coordinates of the mass center. In this example symmetrical mass distribution was considered, thus $y_m = 0$ and $z_m = 0$, $\Theta = m(a^2 + b^2)/12 = 4.128 \times 10^7$ kg m^2/m. The approximate values of the first three circular frequencies, and the results of the ETABS calculation are given in Table 8. The maximum error is less then 12%.

9. Discussion

We presented the stiffnesses of the replacement beam of the stiffening system of building structures. By using an energy approach we derived formulas which show the contribution of the stiffnesses of the individual lateral load-resisting subsystems to the overall stiffnesses of the structure. We took the shear deformation not only in the in-plane problem but also in torsion into account.

Numerical examples were presented to show the usefulness of the replacement beam.

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